Symbolic Polytopes for Quantitative Interpolation and Verification

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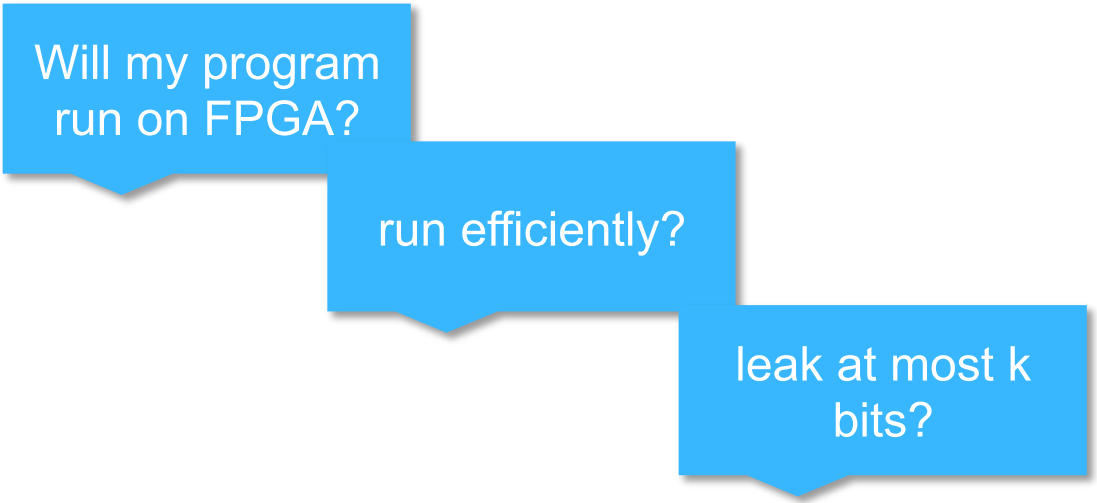
Verification

Will my program crash?

violate API usage rules?

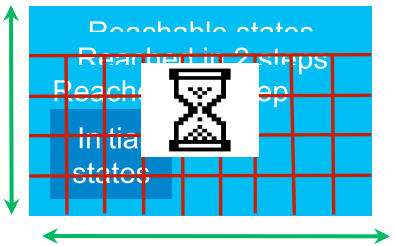
leak secrets?

Quantitative verification



Quantitative reachability property

To prove: at most k states are reachable

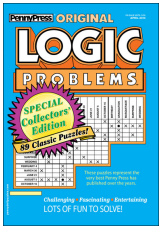


Observables, call-sites, memory locations

$$\text{size}(\text{Reachable states}) \leq k$$

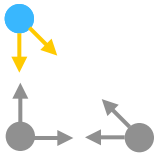
This talk

- Constraint solving problem



- Quantitative interpolation

- Symbolic polytopes and generating functions



$$P(x) = \frac{1}{1-x} + \cdots$$

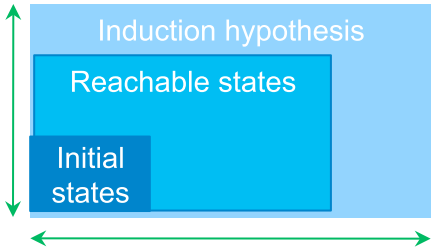
Program as formula

```
int v;

main(int k) {

  init(v,k);

  while (1) step(v);

}
```

$$init(v, k)$$

$$step(v, v')$$

Quantitative verification is a logic problem



Find h:

Base case

$$init(v, k) \rightarrow h(v, k)$$

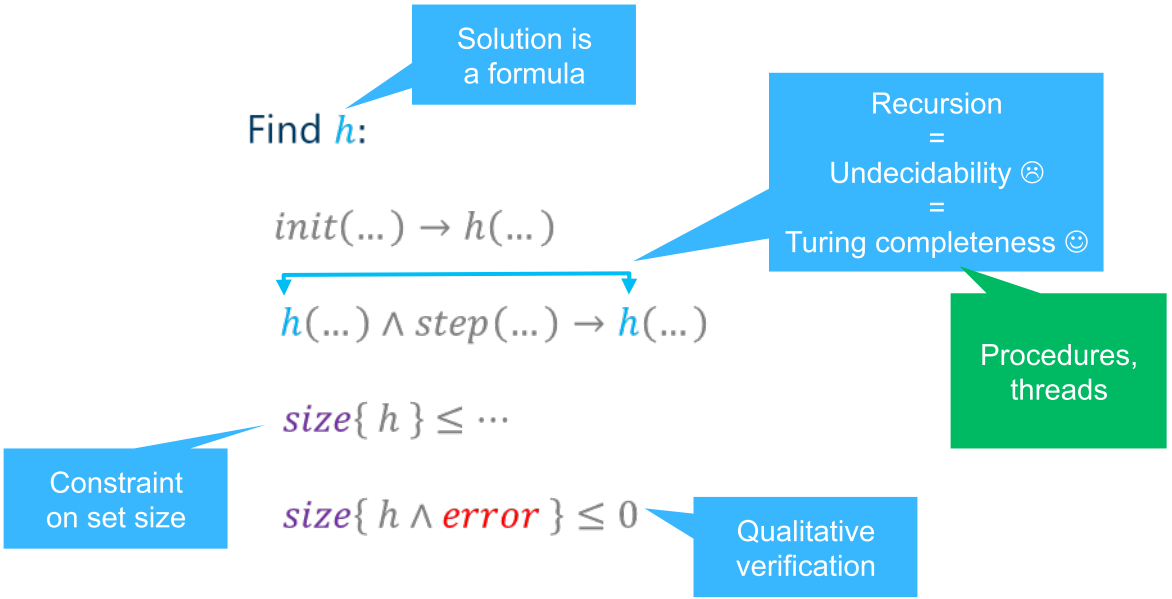
Induction step

$$h(v, k) \wedge step(v, v') \rightarrow h(v', k)$$

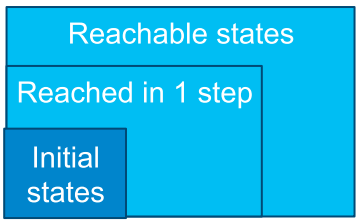
$$size\{\, v \mid h(v, k) \,\} \leq k$$

Sufficient strength

Simple yet expressive



Unfold, guess, and check



Find $h = h_0 \cup h_1$

Bounded problem

$init(\ldots) \rightarrow h_0(\ldots)$

$h_0(\ldots) \wedge step(\ldots) \rightarrow h_1(\ldots)$

$size\{h\} \leq k$

Recursion check

$h(\ldots) \wedge step(\ldots) \rightarrow h(\ldots)$

Solution to bounded problem

If implication fails, increase the bound

Bounded problem is interpolation

[Craig'57, McMillan'03]

Find h:

$low(\dots) \rightarrow h(\dots)$

$h(\dots) \rightarrow hi(\dots)$

Quantitative extension:

Find h:

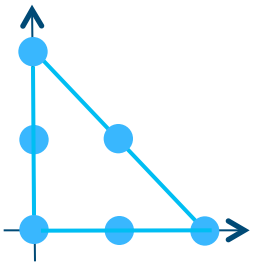
$low(\dots) \rightarrow h(\dots)$

$size\{\, h \,\} \leq k$

From size to set

Interpolants as polytopes

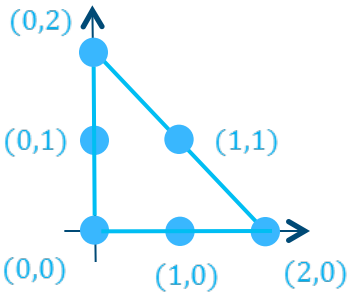
- Integer points represented by linear inequalities
- Model for numeric data types



$$0 \leq x \wedge 0 \leq y \wedge x + y \leq 2$$

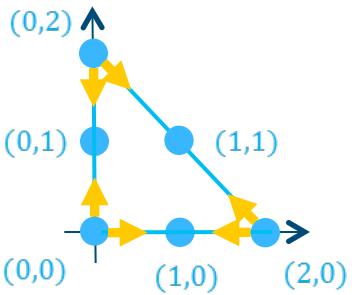
$$size\{\ (x, y) \mid \ldots\ \} = 6$$

Generating functions



- Generating function
  $P(i,j) = i^0 j^2 + i^0 j^1 + \cdots$
- One term per point,
  $P(1,1) = 6 =$ size

Decomposition [Brion'88, Barvinok'93]

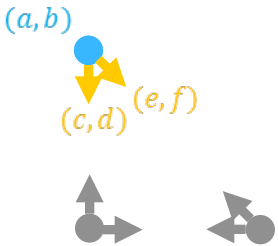


- Rational function

$$P(i,j) = \frac{i^0 j^2}{(1 - i^0 j^{-1})(1 - i^1 j^{-1})} + \cdots$$

- $P(1,1) = 6 =$ size
- One term per vertex

Quantitative interpolation w/o size



$$P(i,j) = \frac{i^a j^b}{(1 - i^c j^d)(1 - i^e j^f)} + \cdots$$

Find h given by a,b,c,d,e,f,... :

$$low(\ldots) \rightarrow h(\ldots)$$

$$P(1,1) \leq k$$

Evaluation

Program	Bound	Time
Dis1 [21]	$max(n - x_0, 0) + max(m - y_0, 0)$	0.19s
Dis2 [21]	$n - x_0 + m - z_0$	0.17s
SimpleSingle [21]	n	0.11s
SequentialSingle [21]	n	0.11s
NestedSingle [21]	$n + 1$	0.15s
SimpleSingle2 [21]	$max(n, m)$	0.13s
SimpleMultiple [21]	$n + m$	0.16s
NestedMultiple [21]	$max(n - x_0, 0) + max(m - y_0, 0)$	0.08s
SimpleMultipleDep [21]	$n \cdot (m + 1)$	0.15s
NestedMultipleDep [21]	$n \cdot (m + 1)$	0.09s
IsortList [23]	$n^2 \cdot m$	0.19s
LCS [23]	$n \cdot x$	0.15s
Example 1 [41]	n	0.15s
Sum [24]	$2n + 6$	0.15s
Flatten [24]	$8l + 8$	0.13s

Find h:

$init(\ldots) \rightarrow h(\ldots)$

$h(\ldots) \wedge step(\ldots) \rightarrow h(\ldots)$ 

$size\{\, h \,\} \leq k$

recursive

$low(\ldots) \rightarrow h(\ldots)$

$size\{\, h \,\} \leq k$ 

bounded

$P(1,1) \leq k$

generating functions